

# Risk Analyses and Rapid Prototyping

## Monte-Carlo Simulations

Frankfurt

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# Agenda



- The Problem
- A Universal Solution
- Problems of the Solution
- Strategy of the Risk Kit: Supply a Toolkit for Risk Management
- Examples

# The Problem



- Mathematically oriented risk management aims generally at the calculation of a value or loss distribution that reflects risk and makes it quantifiable.
- In a second step, measures are taken to influence this distribution.
- The context of risk is frequently so complex that it is impossible to calculate the loss distribution analytically in acceptable time and with sufficient precision.

# Universal Solution



- According to the theorem of Glivenko-Cantelli, for any distribution  $F$ , the empirical distribution function converges uniformly with probability 1 against the true distribution function.

$$\hat{F}_n(x) := \frac{1}{n} \cdot \#\{1 \leq i \leq n | X_i \leq x\}$$

$$d_n = \sup_x |\hat{F}_n(x) - F(x)|$$

$$P\left(\lim_{n \rightarrow \infty} d_n = 0\right) = 1$$

# Universal Solution



- This implies that any distribution  $F$  can be approximated by its empirical distribution function with arbitrary precision.
  - ⇒ Generate the empirical distribution function through simulation of the risk model.
  - ⇒ Monte-Carlo Simulation

# Problems of the Solution



- A Monte-Carlo-simulation requires:
  - The generation of random variates related to the distributions relevant for the considered model.
  - A quantitative, statistical evaluation of the simulation results.
  - In conclusion: a combination of IT and mathematics.
  
- Therefore, the practical realization of a Monte-Carlo-simulation is:
  - Time-consuming
  - Prone to errors
  - Complicated in parts

## Strategy of the Risk Kit



- The Risk Kit supplies a tool-kit for risk management, which:
  - Encapsulates the complicated parts of Monte-Carlo simulation
  - Facilitates a **rapid prototyping** of possible risk analyses
  - Is user-friendly and presents only few limitations
- It uses Excel as a familiar interface:
  - The user can utilize the Risk Kit on a tool which he already possesses and uses, and on which he already has many existing models, **Excel**.

## Strategy of the Risk Kit



- The user implements his model in Excel with additional cell functions which are supplied by the Risk Kit.
- The Risk Kit turns the model into a Monte-Carlo simulation, then runs this simulation and evaluates it.

# Functionality



## ■ Univariate Distributions:

- Beta
- Burr
- Binomial
- Cauchy
- Chi-Square
- Cosine
- Discrete Uniform
- Double Weibull
- Erlang
- Exponential
- Extreme Value (Gumbel)
- Fatigue Life
- F
- Frechet
- Fisk
- Gamma
- Geometric
- Generalized Beta 2
- Hyperbolic Secant
- Hypergeometric
- Inverse Gauss
- Inverse Normal
- Laplace
- Logistic
- Loglogistic
- Lognormal
- Multinomial

- Negative Binomial
- Normal
- Normal Inverse Gauss
- Pareto
- Poisson
- Power Lognormal
- Rayleigh
- Semicircular
- T
- Triangular
- Tukey
- Uniform
- Vasicek
- Weibull

- Density
- Distribution function
- Inverse distribution function
- Random variates

# Functionality



- Multivariate Distributions:
  - Normal
  - T
  - Normal Inverse Gauss
  - Vasicek
  - Normal Copula
  - T Copula
  - Normal Inverse Gauss Copula
  
  - The Copulas can be used to create multivariate distributions with arbitrary marginal distributions and a controlled dependency structure.
- For each distribution
  - Random vectors
- Stochastic processes
  - Brownian Motion
  - Wiener Process
  - Geometric Brownian Motion
  - Black-Karasinski-Process
  - Cox-Ingersoll-Ross Process
  - Vasicek Process
  - GBM Jump Diffusion
  - BK Jump Diffusion
  - Vasicek Jump Diffusion
  - Definition of arbitrary custom made processes
- For each process
  - Random paths
  - For some: trends and confidence bands

# Functionality



- Fixed Income
  - Interest rate and spread curves
  - Valuation of fixed and floating rate bonds
  - Implied default probabilities
- Portfolio risk analyses
  - Fast Delta Gamma VaR
  - Delta Gamma VaR
- Rating functions for SMEs:
  - 4 x Germany
  - Austria
  - USA
- For each:
  - Ranking
  - Default probabilities

# Functionality



- Graphs
  - Histogram (density)
  - Empirical distribution function
  - Inverse empirical distribution function
  - Processes
- Calibration
  - Beta
  - Binomial
  - Gamma
  - Normal
  - Lognormal
  - Poisson
  - Exponential
  - Extreme Value
  - Cosine
  - Logistic
  - Pareto
- Statistics
  - Probability
  - Minimum
  - Maximum
  - Percentiles
  - Shortfalls
  - Ranges
  - Average value
  - Standard deviation
  - Variance
  - Skewness
  - Kurtosis
  - Coefficient of variation

## Example



- The Context  
A person wishes to invest part of his fortune on the capital market
- The Problem  
Shares are by nature a risky investment
- The Objective  
To stabilize profit development through time
- The Procedure  
Describe the sources of uncertainty
- The Result  
Invest part of the portfolio in hedging instruments and evaluate the consequences
- The Vision  
Use of this procedure on general problem situations

## The context



- A person wishes to invest part of his fortune on the capital market.
- He supplies a fixed amount of money as budget.
- He has the choice to divide the budget onto a number of different investments.
- In this example, he may choose between five equities and put and call options on an index, that is constructed from the equity prices.
- (Derivatives with indices as underlyings: Many assets are intended to be hedged with one instrument.)

# The context



The market situation today

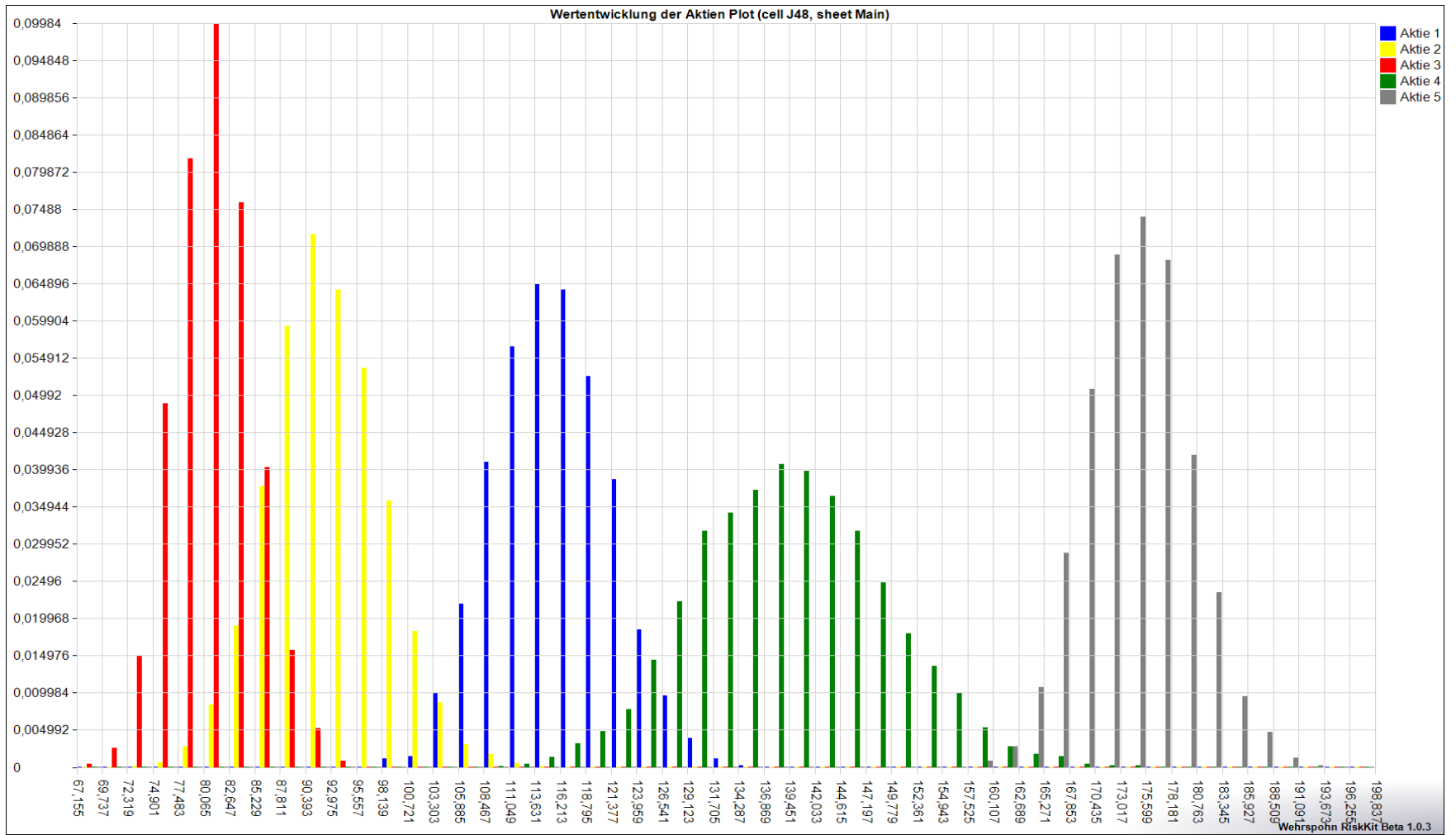
	A	B	C	D	E
1					
2	<b>Marktdaten</b>				
3					
4	<b>Marktparameter</b>		<b>Zeithorizont</b>		
5	<b>r</b>		<b>T</b>		
6	15%		1		
7					
8					
9	<b>Aktie 1</b>	<b>Aktie 2</b>	<b>Aktie 3</b>	<b>Aktie 4</b>	<b>Aktie 5</b>
10					
11	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>
12	100	80	70	120	150
13	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>
14	5,0%	6,0%	5,0%	7,0%	3,0%
15	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>
16	0,25%	0,36%	0,25%	0,49%	0,09%
17					
18	<b>Gewicht im Index</b>				
19	20%	20%	20%	20%	20%
20	<b>Quadrat der Gewichte</b>				
21	0,04	0,04	0,04	0,04	0,04
22			<b>Indexwert heute</b>		
23			104		
24			<b>Indexvola</b>		
25			2,4%		
26					
27	<b>Kursentwicklung</b>				
28	124,24	97,52	79,08	140,86	172,35
29					
30					
31			<b>Indexentwicklung</b>		
32			122,81		
33					

# The problem



- Equities are a risky investment.
- Equity prices are not stable over time, but change seemingly randomly upwards and downwards.

# The problem



## The objective is...



- To stabilize the return over time and to make it more reliable.
- To avoid or mitigate adverse market movements.

## The procedure



- Describe the sources of uncertainty.
- The main step to fetch the risk from its anonymity and to make it accessible for valuation and management consists in describing the factors that influence the target variable (here the value of the equity portfolio) through probability distributions.
- For the random behavior of equity prices there are countless models known from capital market theory.
- We describe the equity price movements over time through a geometric Brownian motion.

# The procedure

Riskfree interest rate

Volatility of equity prices

Random equity prices at the chosen time horizon

	A	B	C	D	E
1					
2	<b>Marktdaten</b>				
3					
4	<b>Marktparameter</b>		<b>Zeithorizont</b>		
5	r		T		
6	15%		1		
7					
8					
9	<b>Aktie 1</b>	<b>Aktie 2</b>	<b>Aktie 3</b>	<b>Aktie 4</b>	<b>Aktie 5</b>
10					
11	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>
12	100	80	70	120	150
13	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>
14	5,0%	6,0%	5,0%	7,0%	3,0%
15	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>
16	0,25%	0,36%	0,25%	0,49%	0,09%
17					
18	<b>Gewicht im Index</b>				
19	20%	20%	20%	20%	20%
20	<b>Quadrat der Gewichte</b>				
21	0,04	0,04	0,04	0,04	0,04
22			<b>Indexwert heute</b>		
23			104		
24			<b>Indexvola</b>		
25			2,4%		
26					
27	<b>Kursentwicklung</b>				
28	124,24	97,52	79,08	140,86	172,35
29					
30					
31			<b>Indexentwicklung</b>		
32			122,81		
33					

## The result



- Movement of the value of the equity portfolio can be compensated by hedging instruments. The enriched portfolio can be valued.
- The valuation of the hedging instruments can again be done with standard methods.

# The result



	A	B	C	D	E
1					
2	<b>Marktdaten</b>				
3					
4	<b>Marktparameter</b>		<b>Zeithorizont</b>		
5	r		T		
6	15%		1		
7					
8					
9	<b>Aktie 1</b>	<b>Aktie 2</b>	<b>Aktie 3</b>	<b>Aktie 4</b>	<b>Aktie 5</b>
10					
11	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>	<b>Wert heute</b>
12	100	80	70	120	150
13	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>	<b>Vola</b>
14	5,0%	6,0%	5,0%	7,0%	3,0%
15	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>	<b>Varianz</b>
16	0,25%	0,36%	0,25%	0,49%	0,09%
17					
18	<b>Gewicht im Index</b>				
19	20%	20%	20%	20%	20%
20	<b>Quadrat der Gewichte</b>				
21	0,04	0,04	0,04	0,04	0,04
22			<b>Indexwert heute</b>		
23			104		
24			<b>Indexvola</b>		
25			2,4%		
26					
27	<b>Kursentwicklung</b>				
28	124,24	97,52	79,08	140,86	172,35
29					
30					
31			<b>Indexentwicklung</b>		
32			122,81		
33					

Definition of the index

Random value of the index at the chosen time horizon

# The result



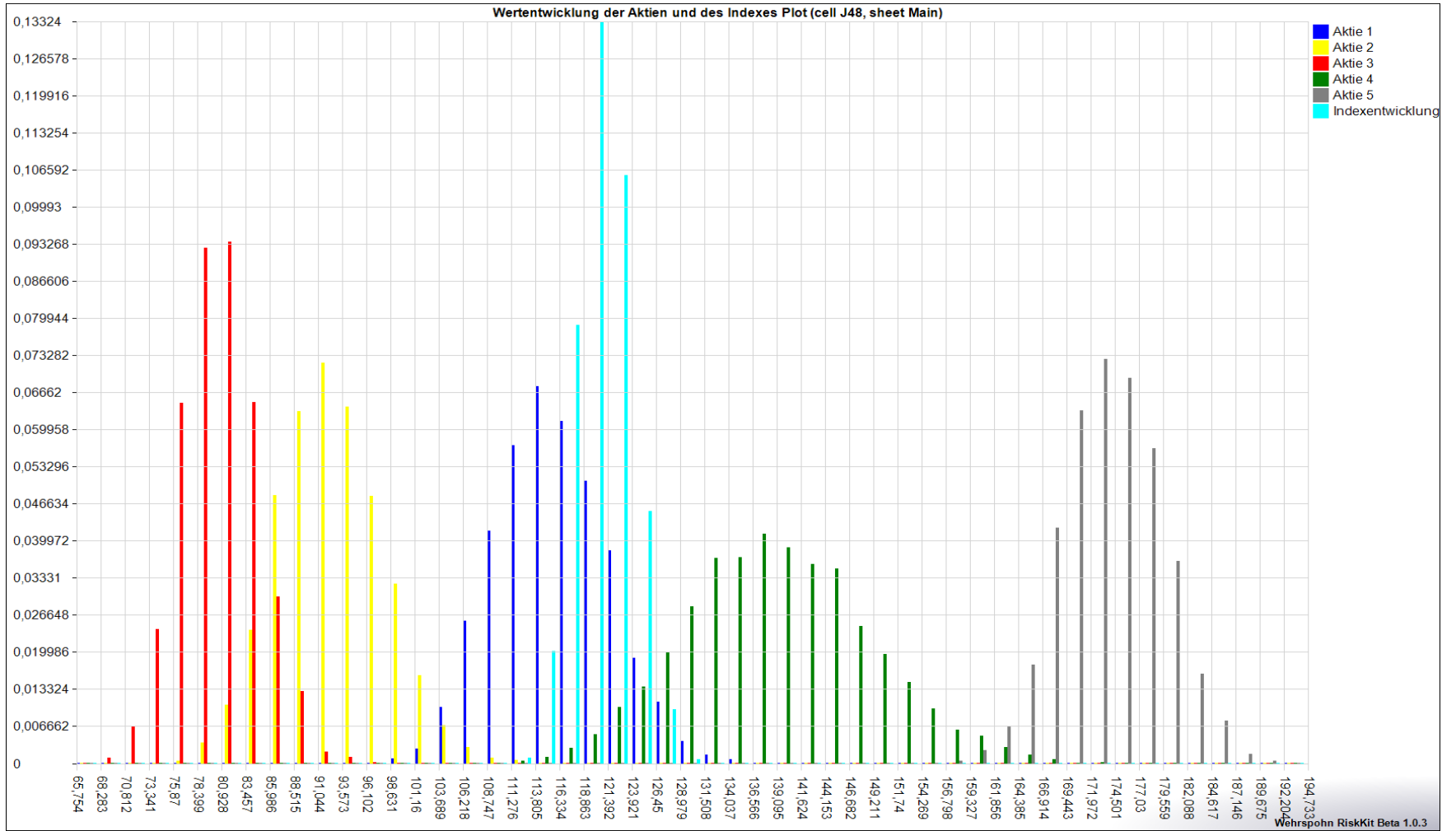
35				
36				
37	<b>Hedging Instrumente</b>			
38				
39	<b>Call-Option auf den Index</b>		<b>Put-Option auf den Index</b>	
40	<b>Strike</b>		<b>Strike</b>	
41	120		120	
42				
43	<b>BS Preis</b>		<b>BS Preis</b>	
44	1,39055216		0,67550933	
45				
46	<b>d1</b>		<b>d1</b>	
47	0,29946485		0,29946485	
48				
49	<b>d2</b>		<b>d2</b>	
50	0,27546485		0,27546485	
51				
52				

# The result

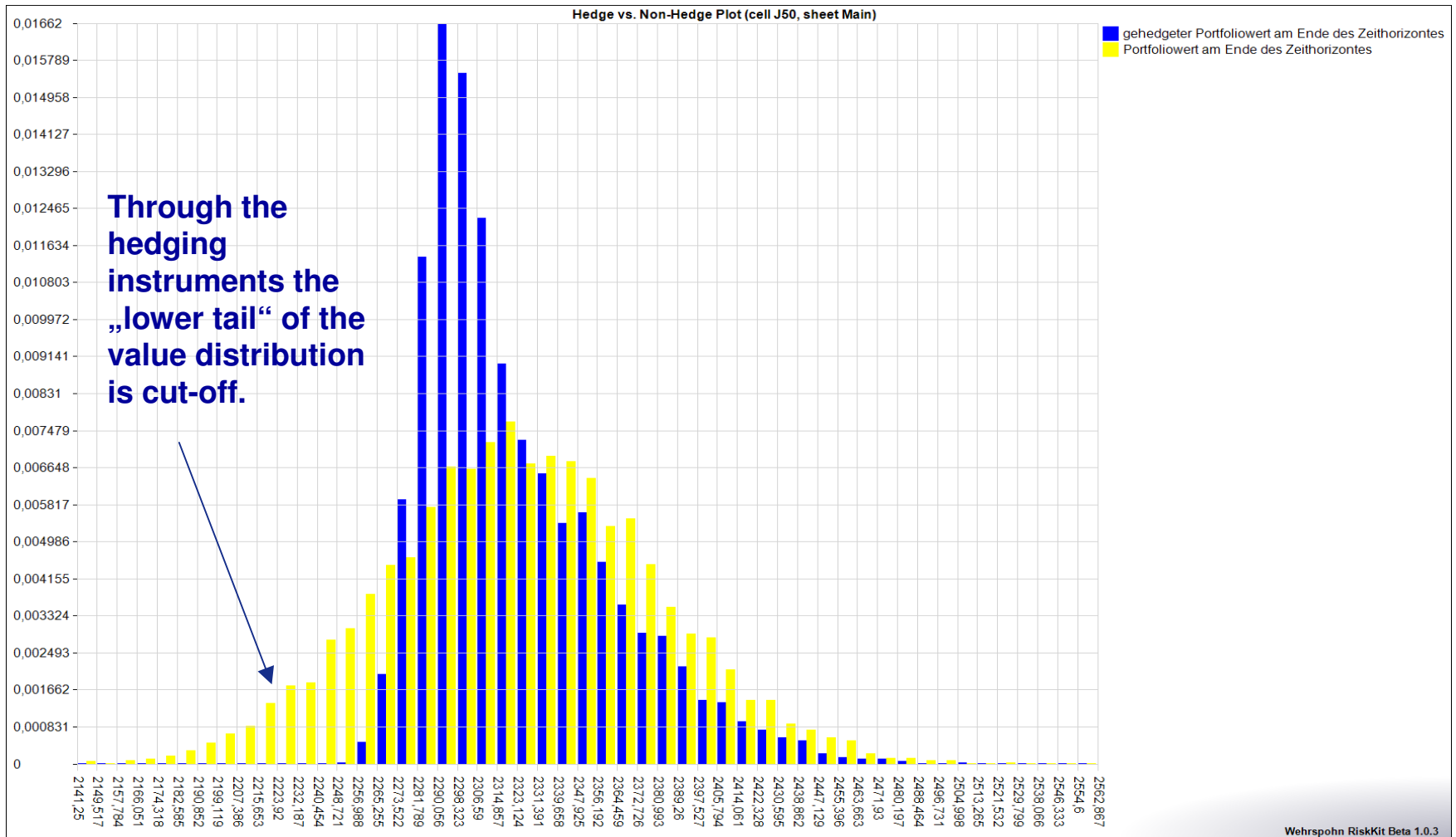


<b>Vermögen heute</b> 2000									
<b>Gehedgtes Portfolio</b>					<b>Ungehedgtes Portfolio</b>				
<b>Anteile am Portfolio</b>					<b>Anteile am Portfolio</b>				
Aktie 1	Aktie 2	Aktie 3	Aktie 4	Aktie 5	Aktie 1	Aktie 2	Aktie 3	Aktie 4	Aktie 5
20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
<b>Call-Option (long)</b> 0					<b>Put-Option (long)</b> 20				
<b>Stückzahlen im Portfolio</b>					<b>Stückzahlen im Portfolio</b>				
Aktie 1	Aktie 2	Aktie 3	Aktie 4	Aktie 5	Aktie 1	Aktie 2	Aktie 3	Aktie 4	Aktie 5
3,97	4,97	5,68	3,31	2,65	4,00	5,00	5,71	3,33	2,67
<b>Aktienwert heute</b> 1.986,49					<b>Aktienwert heute</b> 2.000,00				
<b>Optionen heute</b> 13,51									
<b>Portfoliowert heute</b> 2000,00					<b>Portfoliowert heute</b> 2000,00				
<b>Aktienwert am Ende des Zeithorizontes</b> 2279,68					<b>Aktienwert am Ende des Zeithorizontes</b> 2295,18				
<b>Optionswert am Ende des Zeithorizontes</b> 8,59					<b>Optionswert am Ende des Zeithorizontes</b> 0,00				
<b>gehedgeter Portfoliowert am Ende des Zeithorizontes</b> 2288,27					<b>Portfoliowert am Ende des Zeithorizontes</b> 2295,18				
<b>gehedgtes Portfolio minus reines Aktienportfolio</b> -6,91									

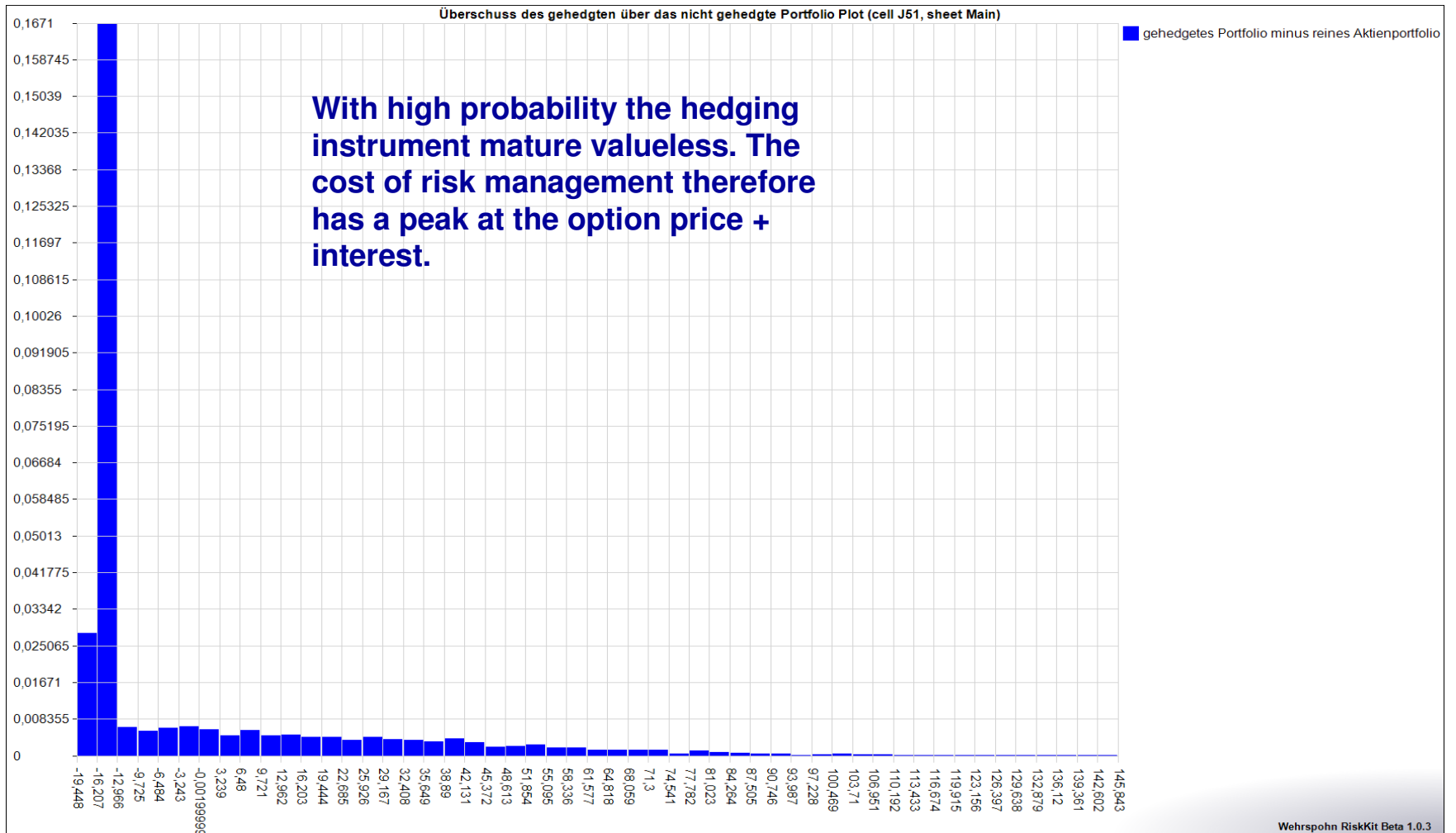
# The result



# The result



# The result



# The vision



- The approach is generally applicable, not only for risks within the capital market.
  
- Examples:
  - Granting of credits
  - Evaluation of projects
  - Decision on investments
  - Insurances
  - Hotel bookings
  - Mergers of companies
  - Sales and marketing
  - ....

# Clients



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